

Simple Equivalence Analysis for Magnetotelluric (MT) Depth Resolution in Layered Earth Model

Disha Ekaputri^{1,2}, Sevi Maulinadya¹, Hendra Grandis^{1*}

¹ *Geophysical Engineering Dept., Faculty of Mining and Petroleum Engineering, Institut Teknologi Bandung, Indonesia*

² *Melbourne School of Engineering, University of Melbourne, Australia*

**Corresponding author: grandis@geoph.itb.ac.id*

Abstract Magnetotellurics (MT) can provide an estimate of subsurface resistivity variation up to a great depth depending on frequency bands of electromagnetic (EM) fields recorded. The skin depth formula represents MT investigation depth, although it is only a rough approximation derived for a homogeneous medium. On the other hand, MT depth resolution is much more complex even for 1D medium. We studied the resolving capability of MT by using simple synthetic 1D models where a thin conductive layer is embedded in a homogeneous more resistive medium. In such case, we consider that the thin layer can be resolved only if it has at least the same conductance (conductivity-thickness product) as the overburden layer. For an overburden with a higher resistivity, a deeper thin layer can be resolved, which is in agreement with the skin depth principle. From the equivalence concept, resistivity and thickness variations of the thin layer with the same conductance can have almost similar model responses. In modeling and interpretation, extremely thin layer should be considered less probable. Basically, MT resolution decreases with depth due to the diffusive character of EM fields.

Keywords: Magnetotellurics; One-Dimension; Resolution; Equivalence; Grid-Search

1. Introduction

Magnetotelluric (MT) sounding is performed by recording natural electromagnetic (EM) fields at wide frequency bands to obtain information of the subsurface resistivity variation with depth. EM fields' diffusive character leads to the well-known skin depth formula used to estimate the MT investigation depth. On the other hand, the decrease of resolution with depth is also an acceptable fact, although there is no explicit equivalent formula for MT depth resolution [1].

Unconstrained 1D inversion modeling may result in very thin layers at depth that are not theoretically nor geologically plausible. Such results often led practitioners and researchers, especially in Indonesia, to promote MT beyond its real capability. The "misconduct" could be avoided if the fundamental concept of the MT method [e.g. 1-3] and the inversion modeling concept in general sense [e.g. 4,5] are well understood. In this paper, we present a simple equivalence analysis using 1D synthetic models to illustrate the resolving capability of the MT method. In principle, for a resolvable thin layer at depth there is no lower bound for resistivity and thickness. Nevertheless, a conservative reasoning can be used to constrain the MT data modeling and interpretation.

2. Skin Depth and Depth Resolution

We consider a three-layer 1D model consisting of a "thin" conductive layer (10 Ohm.m with 100 m thickness) embedded in a more resistive host, i.e. 100 Ohm.m above and below the thin layer. The conductance (conductivity-thickness product) of the thin layer is 10 Siemens. The thin layer's depth or the overburden thickness was varied with 100 m increment from 100 m to 1000 m (model-1). The latter coincides with the overburden's conductance of 10 Siemens or equal to the conductance of the thin layer. Similar exercises were also performed with the host medium of 200, 300, 400 and 500 Ohm.m, but only the latter (model-5) is presented in this paper. In this case, the overburden conductance of 10 Siemens is reached if its thickness is 5000 m, or the thin layer is at 5000 m depth (see Figure 1).

The forward modeling calculations [6,7] were done to obtain sounding curves for model-1 and model-5 with varying thin layer's depth. Deeper thin layer results in a flatter apparent resistivity sounding curve that is more difficult to distinguish from the response of a homogeneous host medium, i.e. constant apparent resistivity represented by dashed curves in Figure 2. We consider that the maximum depth of the thin layer that can still be resolved from MT data is when the overburden's conductance equals to the thin layer's conductance (red curves in Figure 2). In other words, for a thin layer to be resolved from MT data, that layer should have conductance equal or higher than that of overburden

conductance. Figure 2a and Figure 2b also show that for a more resistive overburden, a deeper thin layer can still be resolved, i.e. greater investigation depth according to the skin depth formula.

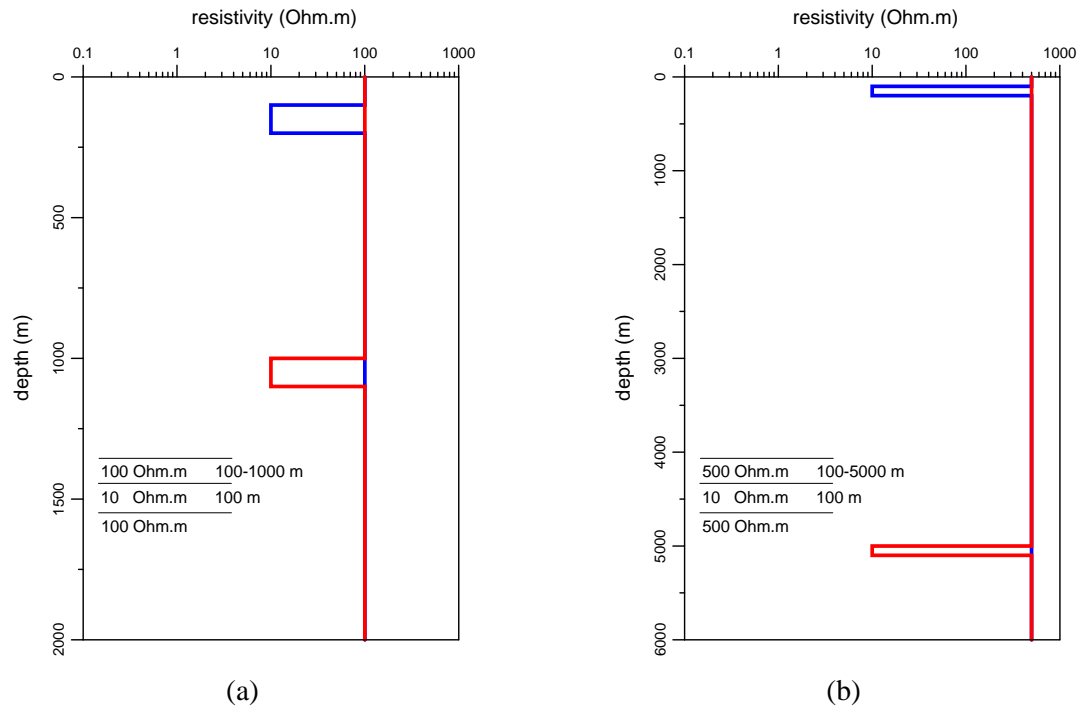


Figure 1. Sounding curves associated with synthetic models for (a) 100 Ohm.m and (b) 500 Ohm.m overburden with thickness varying from 100 m and with 100 m increment. The red curves correspond to thin layer at its maximum depth (see text for details).

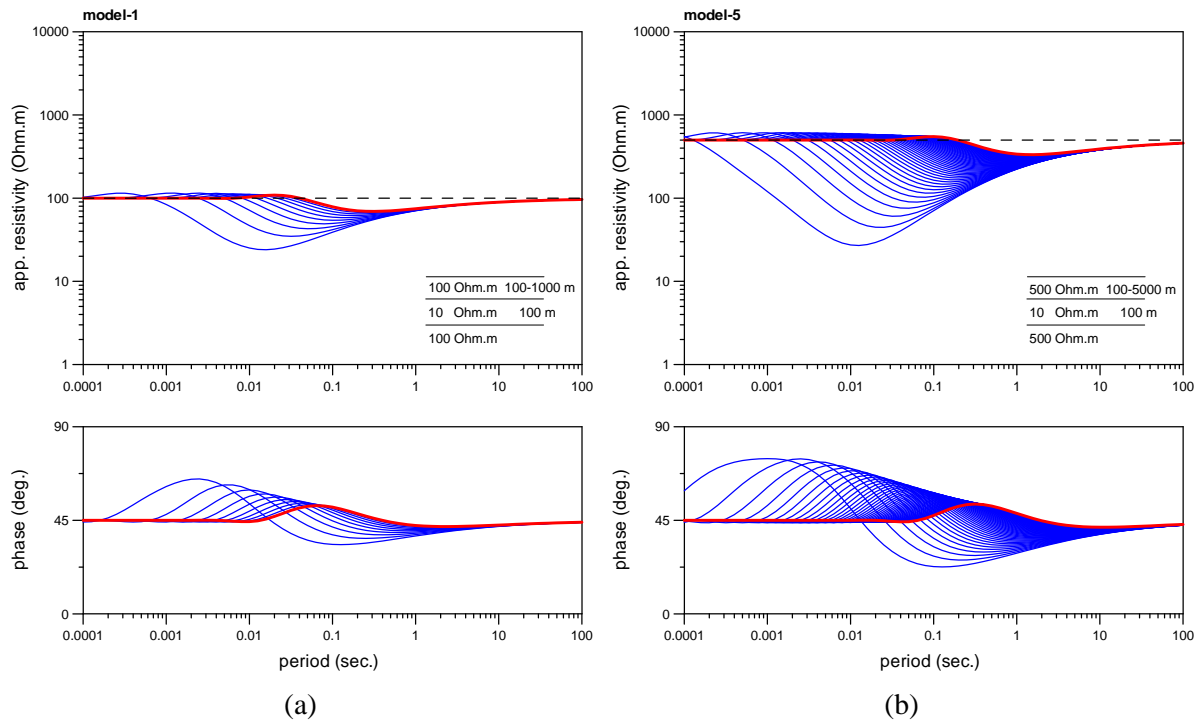


Figure 2. Sounding curves associated with synthetic models for (a) 100 Ohm.m and (b) 500 Ohm.m overburden with thickness varying from 100 m and with 100 m increment. The red curves correspond to thin layer at its maximum depth (see text for details).

We also calculated the deviation of apparent resistivity curves (from the previous exercises) relative to the resistivity of the homogeneous host medium. When the overburden's conductance equals to the thin layer's conductance (i.e. 10 Siemens) the deviations in terms of relative RMS difference are quite consistent for model-1 and model-5, i.e. around 15% with approximately 0.5% uncertainties. This situation is reached when the thin layer is at its maximum depth or the overburden thicknesses are 1000 m and 5000 m for the host medium of 100 Ohm.m and 500 Ohm.m respectively. The deviations are visually represented by deviation of red curves from dashed curves in Figure 2. Smaller deviations than 15% are likely more difficult to discern from the MT data.

3. Equivalence and Thickness Resolution

We performed equivalence analysis to synthetic MT data associated with the thin layer at its maximum depth (the red curves in Figure 1). We used grid search technique where all combinations of the thin layer's resistivity and thickness from the model space are selected, while other parameters (resistivity and thickness of the overburden and resistivity of the third layer) are fixed at their true value. The resistivity from 1 Ohm.m to 1000 Ohm.m and the thickness from 1 m to 1000 m were chosen "a priori" for model parameter intervals. Each parameter model interval covering three decades in the logarithmic scale are discretized into 30 equal sub-intervals homogeneously distributed on the logarithmic scale. Only thin layer's resistivity and thickness combinations leading to 5% RMS apparent resistivity misfit or less are retained. The results are presented in Figure 3 for model-1 and model-5.

From Figure 3 it is obvious that there is a tendency that resistivities and thicknesses for the thin layer are linearly correlated (in the logarithmic scale). This indicates the equivalence of the thin layer in terms of its conductance, i.e. similar conductances lead to similar model responses within a certain amount of deviation or misfit, in this case 5% RMS misfit.

The resistivity and thickness combinations for the thin (second) layer at its maximum depth having 5% RMS misfit relative to the synthetic data (red curves in Figure 1) are plotted as 1D models in Figure 4. These models represent the second layer at its maximum depth that can be considered as still resolvable according to the previous analysis. Very thin layers, as thin as 10 m with appropriate resistivity, may lead to similar apparent resistivity sounding curve to their equivalent thicker layers with the same conductance. In fact, there is no lower bound for both resistivity and thickness of the second layer. If the model parameter interval is extended to include very low resistivity and very thin layer, there are still resistivity-thickness combinations whose responses fall within 5% RMS misfit.

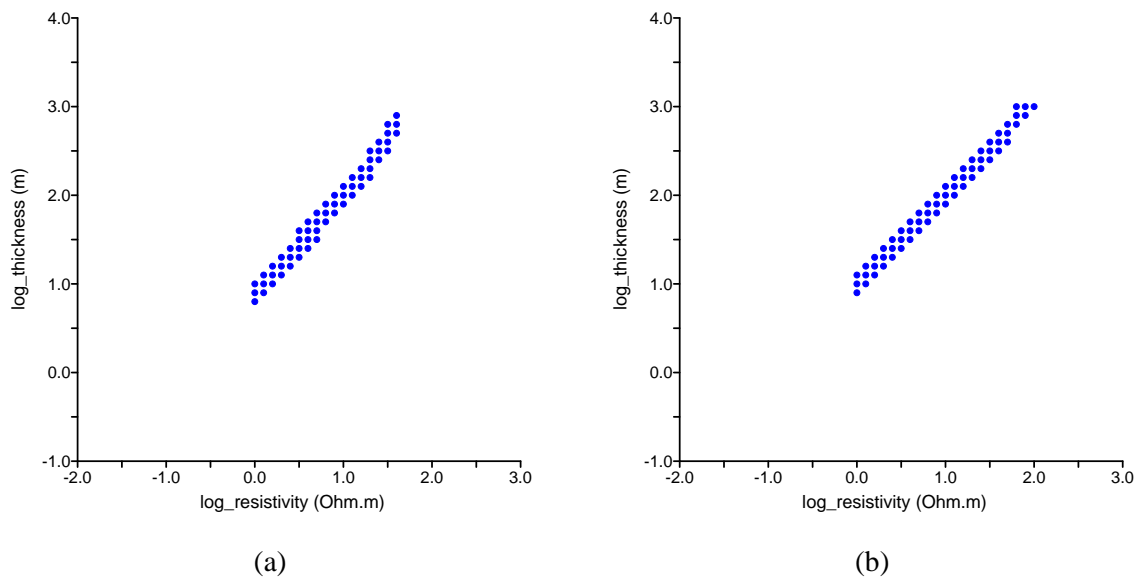


Figure 3. Possible combinations of thin layer's resistivity and thickness with apparent resistivity response within 5% RMS misfit relative to the synthetic data (red curves in Figure 1) for the host medium of (a) 100 Ohm.m and (b) 500 Ohm.m.

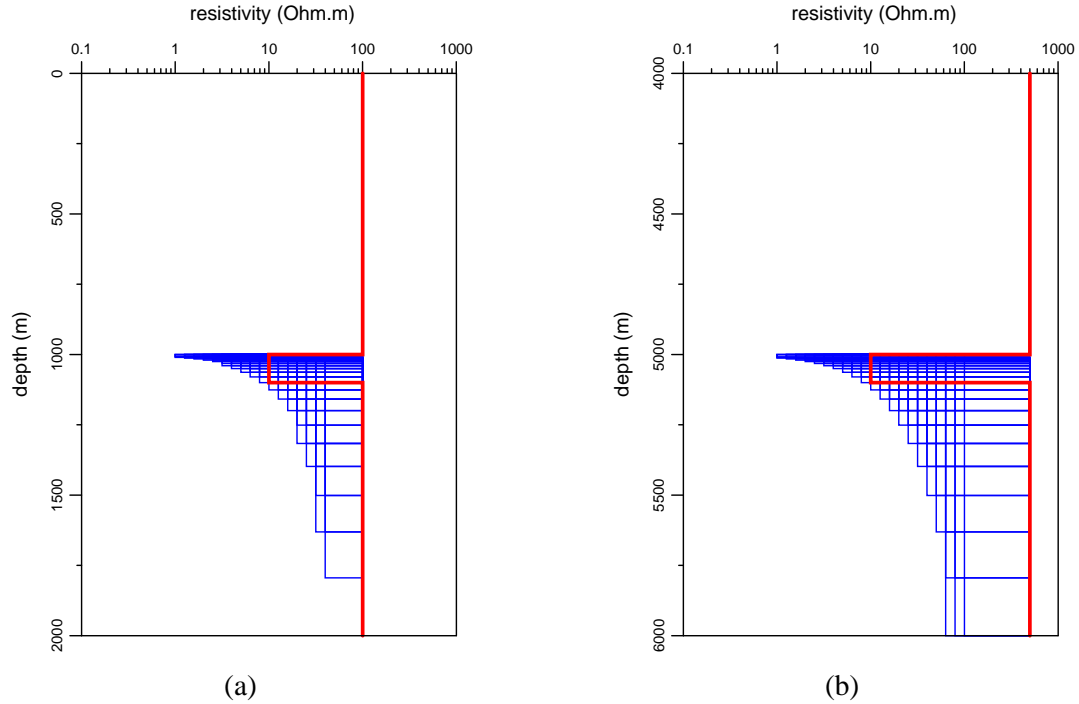


Figure 4. Resistivity versus depth curves representing possible models with apparent resistivity response within 5% RMS misfit relative to the synthetic data (red curves in Figure 1) for the host medium of (a) 100 Ohm.m and (b) 500 Ohm.m.

Apparently, we cannot obtain the minimum thickness resolvable by MT data from previous simple analysis. However, additional "a priori" information can be used to discriminate plausible solutions. In a "normal" situation with reference to a particular local geology and depth range, it would be less likely to have a layer with a resistivity lower than 1 Ohm.m, or it would be impossible to have a layer with thickness less than 10 m or even 50 m. The latter is intuitively inherent to MT or EM methods in general.

From Figure 3 and Figure 4, the range of possible models within 5% RMS relative misfit of the apparent resistivity are larger for deeper second layer (model-5). For a 500 Ohm.m host we can have more resistivity range for the second layer than for 100 Ohm.m host. Therefore, the equivalence problem is relatively more difficult to resolve for the second layer with greater depth. Figure 5 illustrates the ranges of the apparent resistivity curves associated with all possible models in Figure 4.

4. Concluding Remarks

Difficulties in resolving a thin layer at depth from MT data are in fact widely known. However, there are practitioners and even researcher, especially in Indonesia, who try to promote MT method beyond its real capability. Such misconception that is not based on both theory nor reality should be corrected to avoid doubtful results. A simple analysis showed the limitation of the MT method in resolving thin layer at depth simply by plotting the sounding curves associated with 1D models. For a thin layer to be detected by MT data, its conductance (conductivity-thickness product or thickness divided by resistivity) has to be equal or higher than the conductance of the overburden. We have also shown the effect of the overburden resistivity which in a good agreement with the skin depth principle, i.e. deeper investigation depth for higher resistivity.

We have also demonstrated that there are no lower bounds for both resistivity and thickness of a thin layer at depth. In this case, there is no formal and explicit definition of the resolving capability of MT method. Therefore, very thin and very conductive layers at depth may be obtained from unconstrained 1D MT modeling, since they can be associated with similar response to the "true" model. However, from the "conservative" point of view, the MT method cannot resolve an infinitely thin layer although

it is allowed by equivalence principles. Additional "a priori" information can limit plausible models such that solution to 1D MT inverse modeling will be geologically feasible.

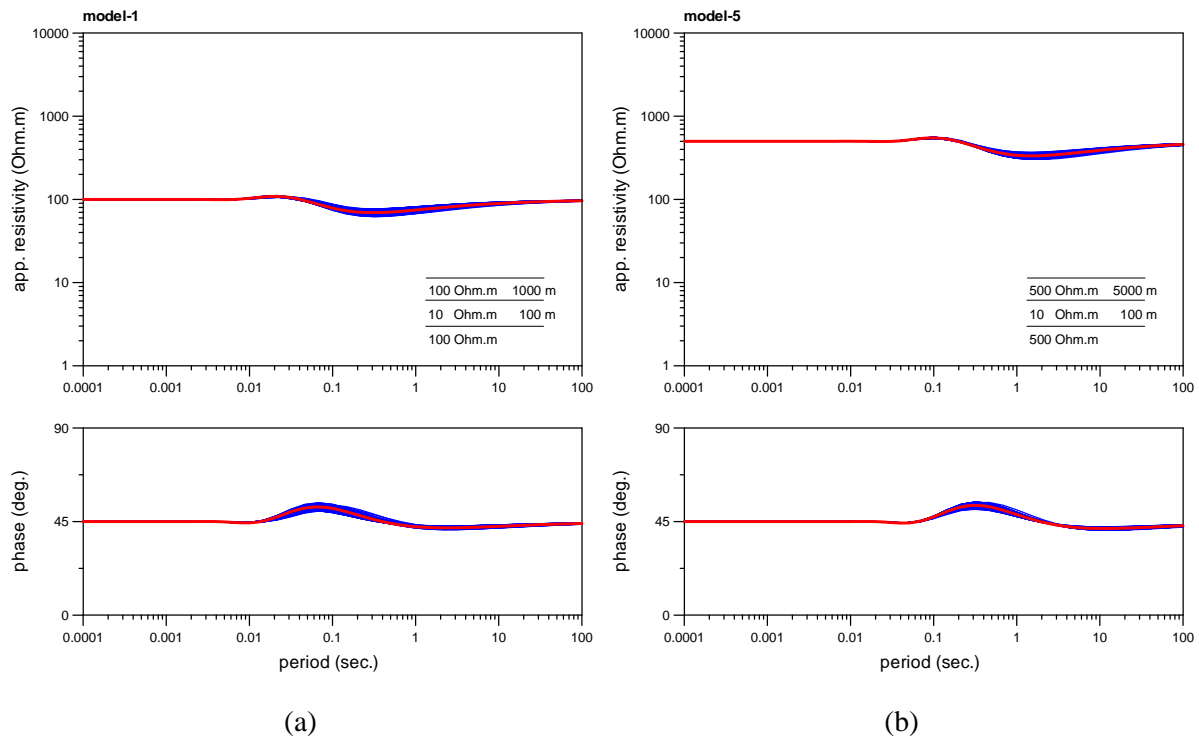


Figure 5. Sounding curves associated with all possible models in Figure 3 for host medium of (a) 100 Ohm.m and (b) 500 Ohm.m. All blue curves are within 5% RMS misfit relative to the red curves (see text for details).

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